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A Method of Reading a Knot

1. Analysis of an example of a knot

The title I have chosen for this chapter indicates which among the less traditional approaches to the problem of knots I am inclined to follow. I would like to show how one can read a knot; if at the same time I help clarify some of the questions addressed by present-day research, this will only be an additional benefit, a by-product of the essential problem I am trying to solve. My basic assumption—that knots lend themselves to a reading—takes me somewhere else: to numbers, letters, graphics and plastic dimensions, which goes against not only currently accepted theories, but in fact all theories of knots (Kaufmann, 1983, 1987).

Changing topology means changing the object, as Quine argues in a different context (Quine, 119). However, it does not mean forgetting classical theory.

The idea that we can read a knot deserves some explanation. I am certainly not saying that the practice of knot-making is a form of writing [*une écriture*]; neither am I trying to argue that a knot is a letter.

Saying that is another matter, which needs to be further clarified before anyone may claim to accept the consequences of the answer I intend to give. In this text, I am not trying to offer a theory of writing.

For now, I would only like to demonstrate that these knots and links are readable, in the same way that we recognize as readable the notches on the bones from Mas-d'Azil, which are now kept in the National Archaeology Museum in Saint Germain-en-Laye.

This stage of readability is essential for writing itself to come into existence, even before we can speak of a constituted form of writing and before we can make any claims as to a specific type of writing in psychoanalysis. Thus reversing the naive order of precedence between writing and reading (Leroi-Gourhan, 1965 and Lacan, *Sem IX,* lesson of 20 dec 1961 and 10 jan 1962), I will be speaking of reading objects, which our modern minds might mistakenly identify with imaginary projections and even with animism. Yet such terms explain nothing—just like before the Freudian discovery of the libido, the word "suggestion" could tell us nothing about hypnosis.

Writing will therefore be another stage, an action of individuals mutually connected by a discourse, by a social bond, who in their actual practice make use of material that is either already available, or some other material, but in any case a material already recovered, a relic of another discourse which has fallen into disuse.

First I would only like to explain that my use of the term reading is not an analogy, as it is often the case—that reading these objects is not the same as reading coffee grounds. In our case, we retain the distinction between calculation and language [*langue*], where we locate the metaphor as a mechanism of signifying condensation based on involution.

However, reading too is an involution of the gaze and the voice. Its structure is clearly seen already in our first section, first in terms of truth and then extended to speech, a formulation which remains a problem for the tired out communicationists. Speech brings us to the knot (Vappereau, 1988 and 1993).

It is apparent that scientific theories of knots are not primarily concerned with the question of reading; the algebraic element of their approach takes it entirely for granted. These theories fail to see that a knot implies an act to be carried out by the subject who is using the object, who fades into a condensation of figures in which he is immersed. They seek to substitute a known [form of] writing [*une écriture*] for the topological body and, taking empirical observation as their model, make no distinction between the two stages—the graphic and the plastic.

Therefore, in terms of the identity of knots discovered thanks to algebraic invariants of standard mathematics, these two aspects—the graphic and the plastic—are hardly at all differentiated.

As algebraic topology, standard mathematics aims to replace a plastic object with an algebraic group[1] or a polynomial[2]; the algebraic object represents a particular case in a vast family of more sophisticated and already known invariants (Kauffman n.d. and 1995). This is my first point.

Our approach does not confuse the formalisation of an object with its mathematisation. In terms of the formation of utterances, our method differs from the demonstration of a thesis in the formal language of mathematical logic. The cause of this confusion, rather than its result, is the forgetting which is the site of our signifying alienation.

Our formalisation, on the other hand, takes condensation into account, because it is both a graphic formalisation of the diagrams of topological objects and a mathematics of their plasticity. This crucial point is then illustrated by the following examples: the coloring and cutting, the duality of diagrams, Terrasson's graph, regular assemblies, Gordian movements and nodal movement.

Coding [*chiffrage*] has its own history and the absence of a distinction between calculation and language largely accounts for the inertia that prevents one from recognizing the actual gestures involved in these practices. This stage implies a subject, even if he is destined to perish in the process.[i]

Then comes the mathematization stage, if it takes place. A structure is discovered; its prototype is the example of algebraic structures and their role within number theory. We see a conversion here—in the psychoanalytic sense of the term—of a series of indexes into symbols, where the structure functions as a text and context to these elements.

This reading presumes that drawing is the site of an involution between a place (topos) and discourse (logos), as correlations of the gaze and the voice. We consider that such drawing is an operation of a cut which, once the drawing has been flattened, can give it the function of lituraterring, and allows us to write, in the small letters of algebra, the numbers [*chiffres*] we can assign to it or attribute to its singularities and which thus precipitate from it. In practice this fact may not be apparent but if we claim to make use of it, we must neither forget nor fail to recognize it. There are in fact theorems that do take the graphic and plastic qualities of knots into account.

Let us define signifying involution, the object of our topology, as "a copula which unites the identical with the different." (Lacan, *Seminar XIV*, lesson of 15.02.67).

Based on this we will also show, in terms of numbers and algebra, what remains unaccounted for in this graphic diagram—namely the problem of non-alterable objects—but can be covered once we finally isolate the plastic dimension, in other words, what is forgotten but insists through its plastic presence and in this way demonstrates the main topological difficulty of all future theories of the knot.

Having underscored the difference between formalization and mathematization, I must also emphasize the existence of a "structure chart" in this approach to involution, which Lacan discusses (Lacan, "Direction of the Treatment," 60/[75]) in connection to the historically crucial example of Newton's law of gravity.

Newton's formula cannot be understood, yet it is explanatory, illuminating and above all it is a solution. Lacan uses it to introduce the littoral function of the letter and to point out its effects of retroactive disruption (Lacan, "The Signification of the Phallus"). We understand that at the extreme it is neither the trace, nor the imprint that upholds the metaphor of the letter, which Lacan is using at this time, and of the practice of reading in psychoanalysis. This practice should be understood as mathematical, between the praxis of the Delphic oracle and Champollion's method.

In order to connect this issue of the handling of utterances [*modes de tenue des énoncés*] with what interests us here, we will take the most easily accessible aspect; however, this should again should warn us against relying on gross analogies. In the register of materiality, let us show that links and knots offer a practice covering the whole spectrum of writing.

This spectrum ranges from mathemes to poems. If we recognize signifying involution as its organizing principle, the two poles find themselves connected, from the simple use of the letter in logic to the practice of calligraphy. Once he has glimpsed this, Wittgenstein immediately gets back on the footpath.[ii] This is where this work would like to make a contribution, with a few remarks about the theoretical elaboration made necessary by Lacan's suggestions.

On the side of the matheme, links and knots depend on the handling [*tenue*] of the utterance, of text, of writing, as it is the case with the grammatical notion of the well-formed proposition in symbolic logic. However, as we have already argued, this also means that, if one is not careful, the reflection of meaning may easily be concealed. As we see in the concept of assemblies in set theory, when used rigorously, this handling can be taken very far. In this case it is the handling that is commonly masked by meaning, as evidenced by the authors who sign their books as N. Bourbaki. These assemblies do not designate sets but are themselves sets (Lacan, *Seminar XX, Encore*, 47-48/[46-47]). Here we will be speaking of a strictly mathematical use of the letter.

For example in Volume I of Bourbaki's "Set Theory," the character designating the empty set: ∅ is therefore

$$\text{Ø est donc } \tau \qquad \in \tau \;\; \in \;\; \square\square\square$$

The reasons why this rigorous characteristics gets little attention have to do with the prohibition on the existence of the structure itself. We can therefore come back to the link between intuition, not just mathematical but also philosophical, and the handling of utterances which have yet to be written.

The quality of a knot and, more specifically, of the Borromean linknot [*chaînœud*], will, contrary to other links, have this function of handling or holding together, but this is not enough. We mustn't forget that between the utterance and the act of uttering, between the object of language and of metalanguage, this manner of handling depends on a subject, yet it can always be formalized all the way to his destitution.

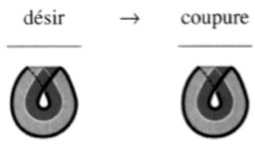
It is true that in practice, this strict use is quickly exhausted, to the point of introducing certain symbols of function. Especially in classical mathematics, with the introduction of the matheme (f: a --> b) which represents its application in set theory (Krivine, 1972; 21). This exhaustion requires other efforts of formalisation but does not repudiate all of them.

Concerning the poem, writing will go as far as to suit the art that is practiced with ink and a brush—the writing of Chinese poetry (Cheng, 1977).

The poem is a function of writing, eminently metaphorical, provided that neither here has it anything to do with analogy, the doctrinal reference of which we find in Lacan's text on the agency of the letter in the unconscious. (Lacan, "Instance of the Letter," 412-441/[493-528]).

As we explain it, starting with the presentation of this series of texts, this function, as it is the normally used in the analytic discourse, whose material aspects it guaranees, reaches as far as the example of the Japanese writing their language,which they had borrowed from China.

Let us take another example of this form of writing, which only applies to the set of results presented in this text.



I argue that the knot can be included in the topological writing of holes, which constitutes the site of the existence of the subject's structure, as an important link that is similar to others and equal in value. This way of writing the *dérive* (drive, *Trieb*), accomplishes what Freud tells us about it (Freud, 1915, "The Unconscious) and what Lacan further clarifies ["Position of the Unconscious," 717-721/[846-850]). It relies on a border, the knot, provided that we also furnish a surface, the libido, which turns out to have a structure, desire, our cut. I have began to theorize this topology of holes and I am going to develop it with the help of the theory of intrinsic surfaces.[3]

As I have said, reading a knot in this way implies assigning it a topological structure, which can be provided by a number theories, the definitions of which are presented in this work. What would we think of a Japanese scholar reading a text written in Japanese, who would claim to ignore the ancient Chinese reading of the letter now used to write today's Japanese? This ancient reading could be dismissed as pure erudition, as supposedly outdated, or foreclosed since Lacan's disappearance; however, as we see particularly well in psychosis—where the foreclosed comes back in the real—in reality, prohibition always remains linked to horror.

I will come back to this practice of reading again in the last chapter, in order to provide a nodal diagram of the clinic of the sinthome, using the Freudian structures of neurosis, perversion, psychosis and analysis, as well as their mutual articulation, which present so many difficulties of reading to the analysands of Freud and of Lacan who lack the topological elements presented here.

If we take this spectrum of variation into account, we see an actual pulsation between the graphic and plastic dimensions of the object. I have already emphasized its invisible presence[4] in connection with masks and tattoos and it also lies at the origin of identification in Freud's understanding of it (Vappereau, 1996).

This is something else than using an image (Eliade, 1952) to try and explain the symbolic function (Frazer, 1981; 652; Leroi-Gourhan, 1965). We must approach the problem by starting from the character of André Gide, as Lacan points out by giving homage to Jean Delay, who in fact discusses the topic at the beginning of his essay on the young Gide. However, we must take this aspect further, as far as we are doing it here ("The Youth of Gide").

This was my second point.

Thirdly, in preparation for this drawing practice, we are first going to create an algorithm, which has previously been lacking, and apply it, until we extract from it a corresponding formula of nodal gravitation.

This algorithm, extended to several rings, is a requirement of this topology, as Lacan stresses in one of the lessons of his Seminar (Lacan, *Seminar XXI*, lesson of 12.03.74) . Here, he is in fact calling for a more rigorous algorithm of a knot, insofar as the latter insterests, as he puts it, more than one ring of string and thus extends, he says further on, Dehn's lemma, which is well known in cases of proper single-ring knots.

At the same time I would like to undertake the task of articulating the question of one and many (Plato, 1967). The thing is that we must pay careful attention to the fact that in addition to this algorithm, in the same lecture, at a specific moment in his teaching, Lacan also refers to having already moved from the Borromean knot (with several rings) to a trefoil (with a single ring). In the seminar of the previous year (Lacan, *Encore*, 122/[111]), we in fact find a brief indication that in order to study the first prime knot, the trefoil, we must refer to the Borromean knot.[5]

This remark carries still more interest once we know that Lacan only used this procedure in the last lecture of his 1979 seminar, in December (Lacan, *Seminar XXVII, Dissolution*), before dissolving his School in January 1980. We do not know whether he had ever explicitly defined this movement [from one to the other]. However, we are now going to construct it with the help of nodal movement[6] and using the tools I are now going to present.

It is also curious and noteworthy that the above-mentioned lecture of the seminar (Encore, lesson of 15.05.73, 122-124/[111-113]) uses the same outline and presents the same objects as a chapter on knots in one particular mathematical treatise (Steinhaus, 1964; 261-268).

I am now going to explain the terminology that we are going to use, so as to begin discussing our topic. When studying the embedding of several rings, we will speak of a link [*chaîne*]. When studying the embedding of a single ring, we will speak of a proper knot, in order to follow Conway's terminology (Conway 1970).

This specification is important becquse our analysis will show that there are links with constant cuts. We are going to call these types of links improper knots or linknots. Whenever we will be dealing with links or knots indifferently, we are going to speak simply of objects.

In the following section, we are going to start by formulating the algorithm predicted by Lacan.

a1—Preliminary remarks

We are working with diagrams [*présentations*] of knots and links, flattened in general position[7], which we are going to call flat schemes [*schémas plats*] S.

Fig. 1:

In the general case, a diagram is non-alternating.

Alternation of a diagram

We say that a diagram is alternating if, in order to pass through all of its components, one after the other, each of the strands of a string moves alternately under, after it had passed over, and over, after it had passed under, the elements of the rings of string through which it is running.

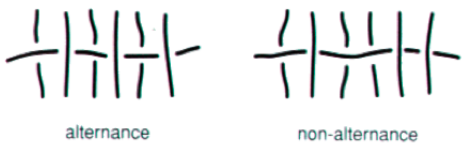


Fig. 2: alternation and non-alternation

In the opposite case we speak of a non-alternating diagram.

For any object in a given diagram, if the diagram is itself not alternating, we cannot be sure that an alternating diagram exists. There are thus alternable and non-alternable objects.

Looking at the flat scheme S, one may be tempted, in order to encrypt the alternance, to simply mark the crossings where a component passes over the elements of the string with a plus sign (+) and where it passes under with a minus sign (–).

Fig. 3

However, the irony of this structure lies in the fact that in encrypting all the components of an object in this way—and in the case of the knot this is done starting from the first component—we find that all the crossings will eventually marked in the same way: by both signs, + and –.

Fig. 4

We need a way of encrypting this alternation that would show the coherency of this specificity and would therefore enable us to explain, thanks to our method, the nature of this distinction.

The Freudian Encryption

Our approach is therefore different from, and we could say even contrary to, this first intuitive attempt.

The resulting encryption is specifically Freudian, in the sense that in order for him to calculate intuitively in this way, the one to discover the Ucs had to be Freud—think of the interpretation he gives of the dream of the "intelligent butcher's wife" and contradicted his own theory of dreams. Our concern is not to find out how did Freud arrive at his interpretation; we only need to recognize it, in order to understand what psychoanalysis depends on. The analysts`s desire carries with itself this unknown.

Starting from the flat scheme S, let us encrypt the alternation by marking the first crossing with a sign of our choice, for example the plus sign (+).

Then, for each component, starting from the crossings already marked:

- we place the same sign as what precedes it on top of the following crossing, if this component passes from one crossing to the other in an alternating manner,

- and put the opposite sign to the previous one at the following crossing if the component crosses it in non-alternating manner.

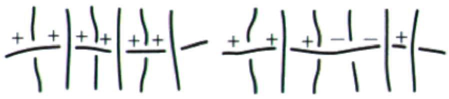


Fig. 5

In other words we are applying an encryption principle which can be formulated as follows:

When the elements of the string are alternating, we do not alternate the signs; when they are non-alternating, we alternate the signs.

This can be seen even more clearly in the following fragment.

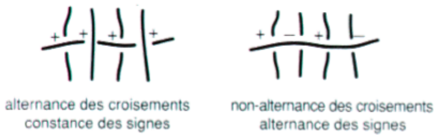


Fig. 6

I call this type of encryption a Freudian encryption.

Let us apply it to the same example, beginning as follows:

Fig. 7

Once completed, this gives us the following result:

Fig. 8

where you can see that among the crossings there are two halves which are themselves alternating but not necessarily related to each other, or, if you like, two sorts of crossings: those marked by a plus (+) and those marked by a minus (–).

Now we are going to introduce a new orientation[8] into the field of these diagrams, in order to account for this phenomenon and to make sense of the encryption, which, for the moment, only exists through its graphic significance.

a2—The knot of 23 July 1993

Fig. 9: Diagram of a non-alternating link, flat scheme.[9]

a3—Analysis

The following three images show the main steps of the analysis we are going to carry out with the help of our algorithm, using colors, for each knot and each link.

Fig. 10

First step: the spanning surface of the given diagram

Second step: the spanning surface is not orientable

Third step: the cut which orients the surface

The colors we are using are represented here by the use of fixed plotting. Their respective functions will become clear in the course of the different stages.

2. The Three Steps of the Algorithm

Let us now look at the first and very simple coloring to be completed in a study of any flattened knot or link.

2.1. Step One: The Spanning Surface

This defines the spanning surface of a given diagram.

Fig 11: First step: The spanning surface of a given diagram.

a1—The goal of this step

We are trying to show a surface in the diagram of a flattened object. This must be a real surface without any folding.

The folds appear as half-twists of the strands. We obtain a compression of the plane.

a2—Carrying out the procedure

Using a binary pair of signs, we go through the whole diagram and label all zones, moving along the free section of each part of an arc[10] and alternating between the two signs passing from one part to the other. This movement runs through the middle of each part of the arc, avoiding the crossings and their vicinity.

All the adjacent zones of the flattened object are then labeled with opposite signs, keeping in mind that two adjacent zones are separated by one part of the arc.

Fig. 12: Adjacent zones in a flat schema

In order to define the spanning surface in the example of our object, we take a couple of signs, such as (+, –) or (0, 1), or (white, grey) or any other couple of distinct and opposite signs which one might use as raw differential elements. We begin by placing one of the signs in any given zone, using the binary (0, 1).

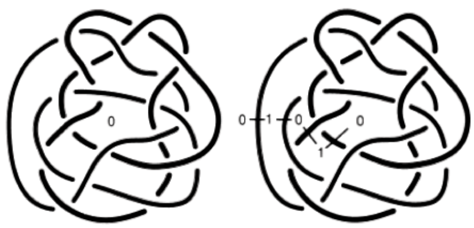


Fig. 13

We write the sign 0 in the first zone. We should cross freely through the middle of a single part of the arc into an adjacent zone. The new zone will be labeled 1. Then, from this zone labeled 1, we move into another one by crossing another part of the arc, and label it 0.

We continue from one zone to another, always crossing the section of the arc in the same way, staying clear of the crossings, until all zones have been labeled with a sign (0 or 1).

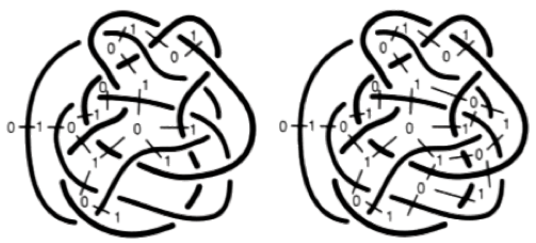


Fig. 14

Note that this algorithm never results in a contradictory situation: the same zone will never be labeled with two opposite signs; two parts of the same section of the arc will never have the same sign, as confirmed by Jordan's theory of plane curves.

We have thus obtained two distinct sets of planes: those labeled "0" and those labeled "1."

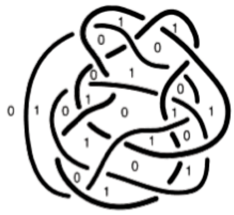


Fig. 15

End of the algorithmic procedure.

a3—Assessing the result

Now we adopt a terminological principle which will enable us to define the spanning surface of a diagram.

The spanning surface of a diagram

We agree on the following:

The set of zones labeled with the sign of a peripheral zone is the set of the empty zones of a given diagram.

Consequently, we define the set of the full zones of this diagram as the set of zones carrying a sign opposite to that of the peripheral zone.

According to this rule, the set of full zones which are connected by half-twists define the spanning surface of the diagram.

In order to emphasize it, we color this surface in: the knot or link now looks like a deformed checkerboard.

Fig. 16: The spanning surface of a given diagram.

We label the number of full zones P (here, P = 11) and the number of empty zones V (V = 10), not forgetting the exterior zone.

The first step of the algorithm is now finished.

However, if we label C the number of crossings, we arrive at a formula derived from Euler-Poincaré's characteristic of a sphere[11], as a result of the sphere's tessellation by the graph of full zones or its dual, the graph of empty zones [12].

This formula tells us that on a sphere, the number of full zones (apices of the graph of full zones), minus the number of crossings (bridges of the full zone graph), plus the number of empty zones (sides of the full-zone graph) always equals 2.

This can be written as:

$$P - C + V = 2$$

which we can then transform using a simple calculation resembling arithmetical calculation, which is quite legitimate since the letters necessarily refer to numbers. Thus:

$$P + V = C + 2$$

which gives us what we will call the elementary formula of a knot: $C = P + V - 2$.

Or in our general case: $C = 11 + 10 - 2 = 19$.

a4—The case of an alternating diagram

In alternating cases, in their alternating diagram, the minimum number of crossings allows us to find the diagram with minimum spanning surface.

In such cases, we designate the set of the more numerous zones as the full zones of the minimum spanning surface and the set of the less numerous as the empty zones.

The minimum number of crossings

We know that for each object there exists a diagram with a minimum of crossings; we are going to call these minimal diagrams, although we will not be able to find them for each case.

When the object is alternable, its alternating diagram is minimal. Using his polynomial, L. Kauffman has shown that in the context of the first stage of our algorithm, which serves to determine the spanning surface, the minimum number of crossings is a topological invariant of alternable knots.

In cases where an alternating diagram is found, we can be certain that the studied object is alternable and consequently in its minimal diagram.

If the alternating diagram exists, we are able to determine the graphic type of the object using the colorings produced by the algorithm. Such typology is a just an initial terminological convenience, which then allows us to reveal the object's nodal and plastic structure. These colorings can also be used for non-alternating diagrams and they provide us with valuable information, e.g. for counting the linkings or carrying out transformations.

The minimal spanning surface

In the alternating diagram of an alternable object, the set of the more numerous zones, which have been chosen as full, connected by half-twists, defines the minimal spanning surface. The empty zones must be the less numerous zones.

Fig. 17

Here, the spanning surface of this diagram is the minimal surface, since V = 4 is less than P = 5. However, minimal surface, which is defined by the full zones, does not always match the spanning surface of a given diagram as we have defined it.

Fig. 18

For example here, the surface of this diagram is not the required minimal surface because P = 3 and V = 4.

In order to reverse this relationship and obtain P = 4 and V = 3, there has to be an exchange of the full and empty quality between the two sets of zones defined by the algorithm.

However, the surface we obtain is no longer the spanning surface of the given diagram: it no longer meets the conditions we have set for such surface in its definition. In order for it to be that of a dual diagram, we need a spanning surface that fits such definition.

Let us explain this with the help of precise definitions.

Duality

We call duality the exchange of full and empty zones in a given diagram.[13]

Dual Surface

We speak of surfaces which are dual to each other: two surfaces which can be obtained one from the other by means of duality.

In the case in question, where we are looking for the minimal spanning surface of an alternating diagram, in the presence of this minimal surface, dual to the surface of the given diagram, we must pay careful attention to the definitions.

However, the previous rule which defines the spanning surface of a diagram forces us to change the diagram if we want this dual surface to be the diagram's spanning surface, in order for the empty zones to be of the same sign as the peripheral zone, as indeed the definition requires.

Let us therefore move on to the dual diagram.

Dual Diagram

In order to obtain the dual diagram of a given diagram, all we have to do is to draw around a peripheral arc and fold it over the other side of the figure.

In other words, it suffices to draw a circle around the figure and then connect it to the peripheral arc.

This planar trick, which consists in using a supplementary circle, is in face a change of the diagram. It is really a permanent deformation of the peripheral arc in question.

I will show this procedure on the example we chose at the very beginning.

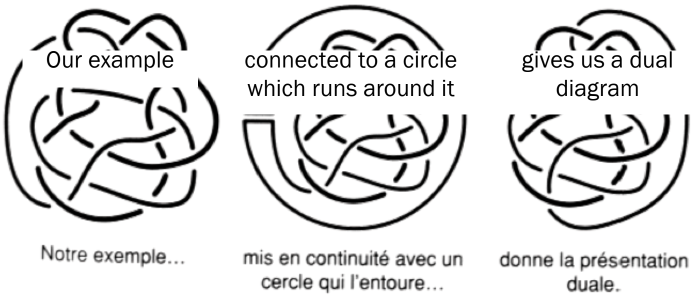


Fig. 19

This change of diagram, which, if we run the deformed arc above and below the figure,[14] involves all the crossings of the diagram, is even more regular in the case of the unpunctured sphere, because there the deformed arc runs along the hidden side of the sphere and does not involve any of the crossings.

This change of diagram can be repeated several times.

In the case of a punctured sphere, our sheet of paper, we are therefore speaking of mutually dual diagrams, according to whether the peripheral zone, the zone which carries the puncture in a sphere when the latter is punctured, is part of either one or the other half of zones as determined by our first algorithmic procedure.

This represents a solution to the question raised by Listing at the end of his habilitation thesis, in which he speaks about different diagrams of the same flattened object. Listing identified this binary system of zones and labeled them λ and θ.

I will come back to this notion, which is very important for our drawings, later in more detail.

Now that we have clarified these definitions, let us go back to the example of the alternating case whose spanning surface we were trying to find, and show that it is the spanning surface of our example's dual diagram, moving from one to the other using the procedure of the supplementary peripheral circle. This may initially appear artificial, but we are going to use it as a practical and graphic definition of the duality of diagrams.

Fig. 20

Looking for the minimal spanning surface in alternable cases when the objects are in their alternating presentation has lead us to change the diagram by using this still slightly enigmatic movement, the duality of diagrams, which will be explained later on.

Arriving at the spanning surface of this diagram, we in fact obtain the minimal spanning surface of the object, V < P,

Fig. 21

because in this case, V = 3 and P = 4. This is indeed the same object, as proven by the change of the diagram.

We may also encounter cases of balanced diagrams.

Balanced Diagrams

We say that a diagram is balanced if P = V.

In such cases, the two spanning surfaces, which are mutually dual, can both be called minimal.

a5—Crumpled surfaces

Readers of Freud may remember what little Hans says about the crumpled giraffe. As Lacan points out (Lacan, *Seminar IV*), if the big giraffe represents the mother, it is easier to sit on the small giraffe drawn on a piece of paper. In this way, he marks the key feature of this observation—that this is no longer the real giraffe. We are now in the Symbolic, which indicates the register of the little boy's naughtiness at this time. Freud highlights this when, at a certain moment in his commentary, he argues that Hans has not yet entered analysis because he has not yet elaborated the register of fiction to which the said naughtiness corresponds.

This dimension of fiction, the dimension of truth, which felt obliged to base on a calculation, is the topic of the first volume, dedicated to logic,[15] of this series of works, which introduce and review the topology and mathematics of the Freudian field.

Going back to the beginnings of psychoanalysis, to the meaning of dreams, we must emphasize the great importance of the optical apparatus described by Freud, in order to detach the reader from a prejudice that remains equally stubborn today—namely that the subject must be located in the mental structure. This is where Lacan begins: his optical scheme is slightly more elaborated, but it can be developed into an analysis of a painting, and not just any painting but Vélasquez' Las Meninas, in order to establish the real lines of the construction of linear perspective. These lines cannot be localised in space, althought they may be reproduced at any given moment. Therefore in order to understand the place of structure, we only need to move on to the virtual objects of our topology, where any given material can only provide us with a local view. Today's computer animations show us a nodal space, inasmuch as it can be calculated by recursive procedures; still, this space remains to be read and to read it we need a reader.

Freud's efforts to explain the dream's rhetoric and its place points towards the necessity of this topology. It would be a very rough approximation to say that the dream is written on a crumpled piece of paper, because it is as if knotted together by the dream work, desire itself; it is written on a libidinal substance, of which the text delivers us the fabric.

2.2. Second Step: Orientability

This determines whether the spanning surface is orientable or non-orientable.

Fig. 22: Second step: the spanning surface is non-orientable.

a1. The aim of this step

We are trying to decide if the surface produced by the previous step is unilateral or bilateral[16]. Let us recall the definition of the orientable (bilateral) or non-orientable (unilateral) properties of a topological surface.

Bilateral: this means that the surface has two sides (like a disk)—it is orientable.

Unilateral: the surface only has one side (like the Moebius strip) and it is non-orientable.

The second algorithmic step results in the formulation of a principle which determines the characteristics of the spanning surface. We will use it to decide on the answer or to verify a result obtained after the use of the algorithm.

a2—The principle resulting from the second step

If there exists at least one empty zone of odd valence, the surface is unilateral. In the opposite case the surface is bilateral: all empty zones are of even valence.

Definition of the valence of zones

Each zone is bordered by a certain number of crossings; this number defines the valence of the zone. We shall call the zones of valence one "loops" (boucles), zones of valence two "stitches" (mailles) and zones of valence three "triskeles" (triskels). Let us note that the valence of a zone also gives us the number of the parts of arc adjacent to it.

We can use this principle right away.

If all the empty zones have an even valence, the surface is bilateral. We color it in using two contrasting shades, one for each side.

Fig. 23

In the opposite case, where there is at least one empty zone of odd valence, the surface is unilateral. We can use hatching to fill it in.

Fig. 24

The parity of the valency of empty zones represents an important property, which our principle uses to determine whether the spanning surface is orientable or not.

Before we deduce the principle from the property, let us formulate the second step of our algorithm.

a3—The method

To do this, we are move through the full zones of the diagram, labelling them with distinct signs. The zones are connected by half-twists. This time we are moving from a full zone to another full zone, passing through these half-twists.

In order to determine the bilateral or unilateral character of the spanning surface, we need to use another binary pair. Let us use (+, −).

Using this new pair, we label the full zones which constitute the spanning surface.

We begin by writing a "+" inside the first full zone:

Fig. 25

We then move through a half-twist and write a “-” in the second full zone.

From here, we pass through another half-twist and write “+” in the following full zone,

Fig. 26

And so on, moving through all the half-twists.

—It is either possible that we find two opposite signs sharing the same zone:

Fig. 27

It may in fact happen that we are forced to move through the same full zone several times, passing through different half-twists. As a result, one full zone will carry with several signs. Moreover, these signs will not necessarily be identical but opposite, in which case we can interrupt the process.

—In the opposite case we have moved through each half-twist at least once and have not found a pair of opposite signs in any one zone.

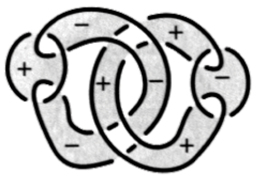
This is the end of the algorithmic procedure.

a4—Assessing the result

We may therefore face two different scenarios.

The first case

There is no opposition. Each full zone carries only identical signs. This is the case in the following example.



La surface d'empan est orientable.

Fig. 28: The spanning surface is orientable.

In this case, the full zones on either sides of each half-twist carry different signs.

The spanning surface is bilateral, there is a + side and a - side.

We will say that the object in question presents itself as an unknot.

Second case

We see that there is a conflict. The algorithm has lead us to put both a + and a - in the same full zone.

This is the case of our chosen example:

La surface d'empan est non orientable.

Fig. 29: The spanning surface is non-orientable.

In this case, all full zones are marked with both a + and a - .

The spanning surface is unilateral, there is only one side.

The object in question presents itself as a knot.

Definition of an object presenting itself as a unknot.

When the spanning surface is bilateral, the object in question presents itself as an unknot, or in other words, its diagram is a diagram of an unknot.

As we have explained earlier, until we have moved through all the half-twists, the unilateral or bilateral nature of the object cannot be determined with certainty. Only on this condition can we be sure that a surface is bilateral.

The diagrams of unknots have a bilateral spanning surface; we mark this using two distinct colors, each for one side.

Fig. 30: The spanning surface is orientable.

Definition of an object presenting itself as a knot

When the spanning surface is bilateral, the object in question presents itself as a knot, or rather, its diagram is the diagram of a knot.

It is possible that the non-orientable character of the surface, which is shown by the opposition of two signs within the same zone, will not be revealed as quickly as in our example. As long as the signs labelling the same zones are homogeneous, we cannot decide on the type of the surface with certainty; it is necessary that we have moved through all the half-twists.

When objects present themselves as knots, their spanning surface is unilateral. We mark it by hatching.

Fig. 31: The spanning surface is non-orientable.

In cases where the surface is unilateral, we can reorient it and make it bilateral. To do this, we simply need to operate a cut. This cut can always be linked and drawn as a cercle, as we will see in the third step.

This is the end of the second step.

a5—Demonstration of the principle deduced from the second step

From the second step of our algorithm we can deduce a principle we have formulated, which helps us determine the bilateral or unilateral characteristic of the spanning surface and thus, in alternating cases, the type of the diagram in question (knot or unknot).

Let us recall the principle we would like to deduce.

If there exists an empty zone of odd valence, the surface is unilateral.

We have defined the valence of a zone as the number of crossings or the number of parts of the arc adjacent to it.

Considering solely the empty zones of our diagram, we are now going to focus on the parity of their valence, as in our example.

Fig. 32: Empty zones of odd valence / empty zones of even valence.

The parity of these numbers has an immediate consequence on our procedure. We notice that we only have to carry out a circular motion from one full zone to another and around an empty zone of odd valence, alternating between + or - each time we are moving through a half-twist, until we return to the initial zone.

Fig. 33

The last and first sign written in the final and initial zone of this cycle will be different because the full motion includes an odd number of passages through the half-twists. This creates an opposition between the two signs that are placed in the same zone.

From this we conclude that if there exists at least one empty zone of odd valence, the spanning surface is unilateral. This is the principle we had previously announced.

In the opposite case, if there are only empty zones of even valence, we never arrive at this opposition and the surface is bilateral.

The second step of the algorithm determines the key characteristic of the classification of surfaces presented in our work on intrinsic topological surfaces.[17]

a6—The case of alternating diagrams

In the case of an alternating diagram of an alternable object, both of the above cases are possible.

If the minimal spanning surface[18] presents the object as an unknot, we will say that this object is an unknot, inasmuch as unknots offer us the purest presentation of the distributions of linking numbers[19].

If the minimal spanning surface presents the object as a knot, we will say that it is a knot in the sense that it contains a knot in the knotting specific to this diagram. The knot will be shown by making the cut that is required to reorient the surface. Our next task is therefore to calculate the specific number of this knotting and the number of the knot it contains.

Balanced diagrams

If the diagram is balanced[20], that is to say if P = V, we should consider both minimal spanning surfaces.

If one of them is bilateral, we classify the object as an unknot and we can then speak of its minimal spanning surface.

If both surfaces are bilateral, we also classify it as an unknot and either of the two mutually dual spanning surfaces can be considered minimal.

If both surfaces are unilateral, we are going to see that they are characterized in the same way by the cut.

Knots and unknots

Amongst all knots and links, which consist of entanglements of one or several rings of string, we thus distinguish, among the alternable cases, between knots and unknots, as two types of objects which are both closer to the truth of a knot as distinct from a linking [*enlacement*].

In a link [*chaîne*], which in this case means an linking, one of the rings passes through the hole of another ring. In a knot, no ring ever passes through another one and when a ring enters into the hole of another ring, it must then also leave it (*Seminar XXII,* lesson of 13.05.75).

This distinction is key in the first step of our procedure: it is the most easily readable and it is shown by our use of coloring and the related commentary. In the following two chapters, I am going to show that the connection between, on the one hand a link and an unknot with two-colored surface, and, on the other hand, a knot and monochrome surfaces, is based on alternate diagrams.

The smallest unknot is a link, a linking.

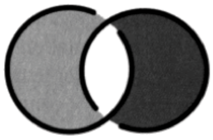


Fig. 34: A simple linking.

Some unknots are made of a single ring. These are proper unknots. The smallest example is the knot Lacan proposes to call "Lacan's knot" (*Seminar XXIII*, lesson of 17.02.76).

Fig. 35: Lacan's knot.

a7—The structure of the libido

Let us establish that the surface characteristic of a fabric depends on the linking and knotting on its border, therefore corresponding to the structure of the drive (Trieb) as described by Freud, where the constant thrust (invariance of the fundamental group[21]) is connected to the source through its border (prevalence of the body orifices, erotogenisation by language).

It was necessary to introduce this surface (quotient of the fundamental group[22]), identified with the libido, as Lacan explains ("Position of the Unconscious" 717/[846]) in order to show this crucial connection in the structure of the Freudian drive. In the movement towards the intrinsic, knotting and linking disappear (s'effacent) like a fold on a fabric, leaving a trace in the form of these characteristics.

The cut, which we can now introduce in the case of monochromatic fabrics, non-orientable crumpled surfaces, traces the path that reveals the structure of the libido. In this way we may understand Lacan's remark (*Seminar XIII*) when he says that we need these non-orientable surfaces, associated with the gaze and the voice, to properly situate desire ("Direction of the Treatment" 502/[601]). Orientable cases, such as the sphere and the torus, are in fact insufficient to account for these connections and Lacan associates them with the objects of pregenital oral and anal drives.

We identify this cut, which condensates the nonorientation of the surface, with desire as metonymy. We can read this at a specific moment in Lacan's teaching, in the involution he makes between metonymy and metaphor, when he comments on

Freud's theorization of the double inscription in his 1915 attempt at a metapsychological work (Freud, "The Unconscious").

Starting from this moment, dream interpretation consists in using the associative material to locate the cut, i.e the main intrinsic characteristic of this fiction of a surface which cannot be found; Freud calls it the libido, i.e. the substance of jouissance which is not there.

This approach will turn out to be still more rigorous, if not exact, based on the number and invariance of cuts, when their number increases due to the number of rings.

2.3. Third Step: The Cut

This step determines the path of cutting which reorients the spanning surface.

Fig. 36: Third step. The cut which reorients the surface.

a1—The goal of this step

In case the surface is unilateral, we can reorient it and make it bilateral. All we have to do is operate a cut.

This cut can always appear as a circle; if it represents several components, these can be connected together.

a2—Carrying out the procedure

In order to decide on the cut, we must choose a new pari of colors. Let us use the following light and dark grey shades:

Using this color binary, we alternate between the two colors, filling in the parts of the arc of each ring, following the successive paths of these rings and putting color on the side of the non-orientable surface, which was produced during the first two steps of the algorithm, as in our example below:

Fig. 37

We begin by coloring a part of the arc, using either of the two colors.

Fig. 38

It is important to note, in order to remain on the side of the spanning surface, that this surface necessitates that we change sides at each crossing. Here we move on to the next part of the arc and therefore must change colors.

Fig. 39

We continue this process until each part of the arc is marked with one color, for each ring we have traced in this way.

Fig. 40

When we are dealing with a proper knot consisting of a single ring, the cut has been identified when the procedure is completed.

When we are dealing with a link, the procedure of coloring each part of the arc of the same ring returns to the starting point, without us having colored the entire surface. We must start again, as many times as it is necessary given the number of rings. We may choose to begin with any part of the arc and use either of the two colors. We complete the task for each ring.

Difference between one and several rings

In the case of a link, the algorithm stops at the first stage—it jams. The procedure of coloring the parts of the arc of the same ring returns to its starting point without having covered the entire surface. We must restart the step, choosing to begin by any part of the arc and using either of the two colors:

Fig. 41

The procedure continues along the second component, moving through its entire length.

Fig. 42

The procedure stops again and we must therefore again move on to another ring, picking a new part of the arc and one of the colors at random.

Fig. 43

We continue until we have traced out the last ring.

Fig. 44

The coloring process has come to an end.

End of the algorithmic procedure.

a3—Assessing the result

We must now interpret the diagram by drawing the cut.

In the fully colored diagram, some of the full zones are monochromatic because all parts of their arc have the same color; other zones are two-colored.

Fig. 45: A full two-colored zone; a full monochromatic zone.

There are two kinds of zones. Full monochromatic zones can be filled in with the same color as the parts of the arcs which function as their borders. Full two-colored zones are held in by crossings, where two parts of the arc of a different color meet in the same full zone. We call these crossings cut crossings [*croisements coupures*].

Fig. 46

We can outline the cut by separating the two colors at the level of each of these crossings by a fragment of the border which lies in the full zone.

By putting these border fragments together we obtain the components of the cut.

The cut runs through each of the full two-colored zones, separating the two colors. This equals to saying that the cut runs around one or more empty zones where all the parts of the arc are of the same color.

Fig. 47: The cut which orients the surface.

Running through full two-colored zones, the cut joins together the cut crossings.

The third step is now finished.

Colorings and orientations

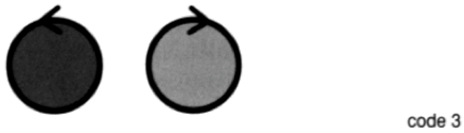
The final coloring strictly corresponds to an orientation of the link-rings and knot-rings[24], according to the following principle of correspondence.

The direction of orientation of a given element of string is shown by the color on the side of this element.

Based on this chosen correspondence, an orientation of the rings which form the boundaries of the fabric may be associated to a given coloring of the fabric.

Fig. 48

Another way of marking the chosen code can usually be extended to the plane of drawings; the boundaries of the zones filled with a given color can be oriented in the corresponding way.



The knot and unknot parts of a diagram

In a given colored diagram, we will call the knot part (the part of the cut) [*partie noeud* (*partie coupure)*] the part composed of the full zone and the crossings through which we have made the cut. This part can have several components.

We will call the unknot part (the non-cut part) [*partie non-noeud* (*partie non-coupure*)] the part composed of monochromatic zones and crossings where no cut has been made. This can also consists of several components.

In the drawings these parts are isolated, by following the outline of a subgraph of Terrasson's graph[25]:

Fig. 49: The knot part and the unknot part.

The sources of these different parts[26] and their mode of composition[27] have been studied in detail.

Having defined the knot and unknot parts of the colored diagram, our algorithm is finished.

a4- Cases of links composed of several rings

In the case of a link comprising several rings, we have seen that the process interrupts itself and we must resume it arbitrarily, choosing a new part of the arc and a new color. A different choice can be made between the two colors, for the part of the arc chosen at the moment of restarting the coloring procedure. These different colorings do not lead to the same result. Therefore, in the case of a link composed of several rings it is possible to make a number of different cuts.

Here is an example based on a general case:

Fig. 50: A cut through seven half-twists and a cut through eleven half-twists.

In the case of links, we therefore have several different ways of making the cut. If we label the number of rings r, the number of possible colorings will be (2r) and the number of cuts (2r–1). These different cuts have the same parity. The theory of intrinsic topological surfaces[28] tells us that this is true because each time we are dealing with the same non-orientable surface, equivalent to a projective plane (in odd cases) or to Klein's bottle (in even cases), and in addition to them a certain number of tori, according to the main theorem of the theory of intrinsic surfaces.

a5—Four interpretations of the dream of "The Butcher's Wife"

Lacan gives us an example of an interpretation of a dream ("The Direction of the Treatment" 518523/[620-627]), which he says he does not do very often but which on this occasion will serve as a paradigm. This is the dream of the "Intelligent Butcher's Wife," transcribed by Freud in his crucial work ("The Interpretation of Dreams," 1900).

Freud's first interpretation is already quite surprising, given that, as we know, the lady in question has brought the dream to her analyst in order to contradict his own theory of dreams, according to which a dream is the fulfillment of a wish. Freud says this upfront—and it really takes Freud, and no-one else, to be able to answer to the beautiful hysteric that her desire is, precisely, to have an unfulfilled desire.

He then pursues his commentary by giving us the first lines of his theory of identification, specifically of hysterical identification, thus adding a second interpretation, which he does not in fact reveal to the butcher's wife. Her desire is to identify with her hysterical friend, who has appeared in the dream's associations, because although the friend is a slim woman, the patient's husband likes her, while full-figured women are usually more to his taste.

Lacan extends the dream's interpretation by giving us a third one, which further elaborates the second. He points out that in her dream, the dreamer also identifies with her husband because she is trying to answer the quintessentially hysterical question by acting like a man: how can a man desire what he does not love?

Finally, Lacan adds that the subject also identifies with the salmon. He speaks about the pieces of gauze separating the slices of smoked fish as an analogy of the veil hiding the phallus, which has just been discovered among the frescoes featuring the demon of modesty on the walls of Villa of the Mysteries in Pompey. This is the fourth interpretation.

How then can we better understand the fact that a dream can have four different interpretations, where each one is equally correct and coherent, if not by using these cuts, which condense the nonorientation of the spanning surface of a link comprised of several rings.

According to the algorithm and the simple calculation I have proposed, three rings can result in four different cuts.

The cut is what the interpretation of the knot should trace: it needn't be exhaustive and pass through all zones; it only has tosum up the nonorientation by reorienting the entire surface, giving direction [*sens*] to the zones of the unknot part through which it doesn't run.

I will discuss the result, dealing with the number of cuts, immediately in the next chapter, in order to interpret the variation in the number of cuts in terms of linkings.

a6—The case of alternating diagrams

In the case of minimal spanning surface of an alternating diagram, we are led to distinguish between two families of knots, as opposed to unknots identified in the second step of our algorithm. These two families are defined according to the parity of the cut.

The cut crosses a certain number of half-twists. We call this number the number of the cut and label it k.

Parity of the cut

We call the parity of the cut the even or odd property of the number of the cut.

If the cut is odd, the alternating knot belongs to the same family as the trefoil.

If the cut is even, the alternating knot belongs to the same family as Listing's knot.

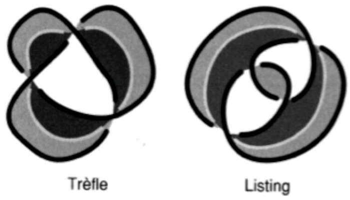


Fig. 51: Trefoil and Listing's knot.

The unknots we have already seen have a zero cut, of the same parity as the Listings[29].

Balanced diagrams

When the diagram is balanced, the uniqueness of the family to which the knot in question belongs, when it is alternable and in its alternating diagram, is also certain. If the two mutually dual spanning surfaces are unilateral, it is easy to show that in balanced cases, the cuts made on one and the other will be of equal parity.

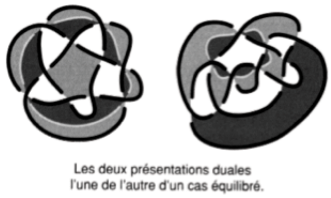


Fig. 52: Two mutually dual diagrams of a balanced case.

Let us now return to the elementary knot formula we established at the end of our first step.[30]

$$P + V = C + 2$$

and recall that, as we defined in the same step, balanced knots are such that P = V.

Under these conditions, the formula becomes:

$$2P = C + 2 \text{ or } 2V = C + 2$$

In this way, it is easy to ascertain that in the particular case of balanced knots, the crossing number is even:

$$C = 2\,(V - 1)$$

The knot part and the unknot part therefore have the same parity because their sum is an even number.

This defines the parity of the cut of both balanced knots and balanced. Such knots and links indeed belong unequivocally to the same family.

3. Conclusion

In the following table, we see the vocabulary adopted, starting from the accepted mathematical distinction between knots, comprised of a single ring, and links, which are constituted by several rings.

For this mathematical criterion, based on the uniqueness or multiplicity of rings, we substitute another distinctive trait, which has to do with the necessity of the cut: whether or not a cut is needed.

We will use the term knot for alternating cases where the cut is necessary, this is to be understood as “a knot exists.”

We will speak of proper knots in a case of a knot made of a single ring, and of improper knots when there are several rings.

We will use the term “unknots” for alternating cases where the minimal spanning surface is two-colored, that is to say it does not require a cut.

<table>
<tr><td colspan="3">Knots (one)</td><td colspan="3">Links (several rings)</td></tr>
<tr><td colspan="2">Cut (knots)</td><td>No cut (unknots)</td><td colspan="2">Cut (knots)</td><td>No cut (unknots)</td></tr>
<tr><td colspan="2">Proper knots</td><td rowspan="3">Lacan’s knots</td><td colspan="2">Improper knots</td><td rowspan="3">Linkings</td></tr>
<tr><td>even</td><td>odd</td><td>even</td><td>odd</td></tr>
<tr><td>Listing</td><td>Trefoil</td><td>Listing</td><td>Trefoil</td></tr>
</table>

Terminology for alternable links and knots of 1, 2 and 3 rings in their minimal alternating diagram.

This terminology is particularly relevant for alternating cases composed of one, two or three rings. In the following part, I am going to explain the reasons for this designation of objects and we will also look at its generalization for a higher number of rings.

In non-alternating cases we adopt a distinction articulated by the phrase “diagram as a knot” in cases where a cut is not necessary, i.e. when there is a coloring that does not require a cut.

The main consequence of these three algorithmic steps is that each proper knot and each alternable link belong to a family of a unique name, which we will use in our description of a variety of knots and alternating links.

This is so because:

- the parity of the cut is set for links comprising multiple rings;
- the parity of the cut is set for proper knots and links with only one and minimal spanning surface (non-balanced knots and links: P > V);

- the parity of the cut is set for proper knots and balanced links (P = V), whichever minimal spanning surface we choose between the two mutually dual spanning surfaces.

Proper and improper knots can be divided into two families, trefoils and Listings.

Unknots can be divided, according to unicity of multiplicity of the number of rings, into Lacan's knots and linkings.

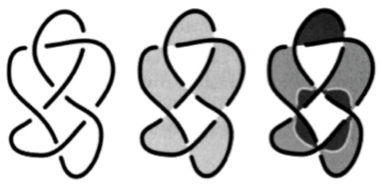
The existence of improper knots among objects normally designated as links deserves some further comments, which I would now like to make by looking at the question of the variation of the cut in cases of multiple ring objects.

4. Exercises

e1—Coloring

In three steps and using a minimum of moves, identify the cut of one knot and one link, when it is necessary.

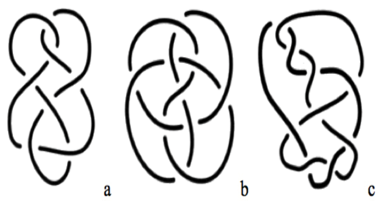
For example, the following are the three steps for the example of knot 62:



la présentation alternée, la surface d'empan, et sa coupure.

Fig. a: Alternating diagram, the spanning surface and the cut.

Do the same exercise for each of the following knots and links:



Be careful with figures b and c. Should you need further explanation, refer to the following exercise.

e2—Making a cut through the folds

1. Transform the drawing of a spanning surface given in this exercise, so that the folding at the level of each half-twist appears clearly[31]. In this exercise, you will be able to see see how the cut runs through the folding and the colors are distributed at the crossings.

Fig. e

2. Find the outline of the cut in the folding and verify that it can run through there twice, in order to join together the two components of the cut of knot 940, the coloring of which you determined in the previous exercise.

Fig. f

Notes

1. See Essaim.
2. See Appendix to Chapter III in this work.
3. See Étoffe, from the diagram of the series on pp. X and XI to the conclusion on pp. 277 to 299.
4. Étoffe, p.41 and Chapter VII, p. 249.
5. See Chapter V.
6. See Chapter VII.
7. See Essaim, pp. 79-88.
8. See Chapter III.
9. Here, we are using a less specific example.
10. The notion of the arcs of a given diagram is defined in Essaim, p. 82. The part of the arc is a piece of the arc between two of its consecutive crossings.
11. See Chapter IV of this work and Étoffe, Chapter III.

12. These graphs consist of vertices placed in the zone of each type, which are connected by edges that run through all the crossings; they are mutually dual. I am discussing these graphs in this work in Chapter IV.
13. See Chapter IV in this work and J-M. Vappereau and M. Bertheux, De la mise à plat et de la dualité des présentations (diagrams) de nœuds ou de chaînes. (Unpublished, Appendix II of this work).
14. See Chapter IV in this work.
15. See Nons, Chapter I.
16. See Étoffe, Chapter III, p. 122.
17. See Étoffe.
18. See above § 2.1.a4 for a definition of this notion.
19. See Chapter V.
20. See above § 2.1.a4.
21. See Essaim, this work deals primarily with the group of knots and links having this property.
22. See Étoffe, Chapter I, p. 60.
23. This cut is a boundary which makes consistent and transforms the surface into a tessellation which can be oriented by pieces, see Étoffe, p. 122, and pp. 134-135.
24. See Appendix to Chapter I.
25. Terrasson's graph connects the vertices placed in all the zones of the diagram by edges that run through all the parts of the arcs in the diagram. It is introduced and used in Chapter VI.
26. See Chapter V
27. See Chapter VI.
28. I refer the reader to Étoffe, Chapter II, where one finds the definitions of a sphere, a torus, a projective plane (cross-cap and Moebius' strip) and Klein's bottle, and in Chapter III of the boundary and the frontier (See the Appendix to Chapter I in this work).
29. In the case of unknots, when they are composed of several rings, we can study even cuts which have the property of making the surface disconnected.
30. See above § 2.1.a3 and Chapter IV.
31. See Étoffe, pp. 62-65, and Chapter III in this work.

Translator's notes

i. Chiffrage is derived from chiffre (figure, numeral, character), which the author situates on the side of writing, of the letter, and thus opposed to nombre (number, digit), on the side of the signifier [Personal communication, 3 march 2013].

ii. See the 9/2/1972 lesson of Lacan's 1971-72 seminar Ou pire… where he comments on Wittgenstein's prohibition in the Tractatus: Whereof one cannot speak, thereof one must be silent. Lacan says: "Hence he could hardly say anything. Every time he would step down from the footpath and into the gutter, he would get back on the footpath, the footpath defined by this imperative." As Jean-Michel Vappereau points out, this is the very opposite of psychoanalysis, "where we stand with both feet in the gutter" [Personal communication, 3 march 2013].

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