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Topological Objects and the Current State of Mathematics

Topological objects, meaning here: knots, chains, braids, surfaces, and others. Topological objects are present in artisanship, in decorative motifs, in puzzles, in mathematics, and in the lectures of Lacan.

Topological objects seem to me to be a bricolage, as possibilities of bricolage. In particular, drawing topological objects, that's a bricolage.

That's not to say that topological objects are made of pieces and bits, of bric-o-brac. Quite the contrary. However, such a point of view exists, under the name "combinatorial topology." This point of view seems to me unsatisfactory because it manages to define topological objects starting from non-topological things; I mean to say, it defines objects that have holes as a set of things without holes. From this point of view, a circle is not a primary object. A circle is defined as a patchwork of various segments. There must be a vicious circle somewhere here.

"Combinatorial topology" is one of the points of view in present day mathematical topology. We remember how topology separates itself from geometry. But today, topology is being erased by something calling itself "general topology."

"General topology" is also called "set-theoretic topology" or "topology of sets of points." As "general topology," a "set of points" is called a "topological space."

I will not use "topological object" in the sense of "topological space" because, on the contrary, I want to oppose topological objects to "general topology." This opposition is indicated, for example, by Fréchet in *Introduction à la Topologie Combinatoire* (1946, pp. 20-22), and also by others.

"General topology" is a theory of infinite sets of points, called "topological spaces." It's full of infinity: there is the infinitesimal, that is to say the infinitely divisible and the infinitely small; there is actual infinity, that is to say that space is conceived of as the coexistence of an infinity of points. What helps deal with this actual infinity is set theory language.

"General topology" is also a theory of boundaries and boundary incidents. It's a theory that refines notions of part and part complement with notions of "interior" and "exterior," and, by doing so, problematizes boundary phenomena.

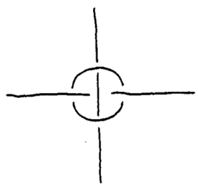
What relationship there is between this double infinity (infinitesimal and actual infinity) and boundary problems, is not clear to me.

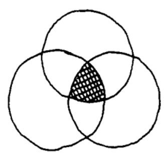
This double infinity, I put it under the heading of "massive infinity." "General topology" is a theory of massive infinity. To make an image of it, I would say that it's like the sea (the coastal sea). I also put it under a formula of indetermination: "zero x infinity" or "0 x ∞.

I've heard Lacan assimilate the body to this thing of "general topology." Body = "topological space."

Massive infinity is different from other infinities. It is different from "repetitive infinity," that is to say, the infinity of a sequence of numbers. It is also different from "topological infinity." "Topological infinity," that's what says that a circle and a line are different, that a sphere and a plane are different.

Lacan has put "topological infinity" into play many times: by introducing chains of lines and circles; by giving the "object a" the status of a plane; by making a reversal of the torus; by situating the couple (desire/demand) on the torus.





With "general topology," space thus has been associated with a sophisticated infinity. But all the same, a big confusion prevails today, because there is a tendency to reduce all spatial consideration to massive infinity. And the finite finds itself defined by the infinite. This is what I put under formulas of indetermination: "infinity—infinity" or "∞" and "infinite / infinite" or "∞/ ∞." And especially, topological objects have nothing to do with massive infinity. Said otherwise, the notion of "hole" has nothing to do with the infinitesimal. Said otherwise, topological objects—that is to say pure topology—have nothing to do with what is called "general topology."

Why has massive infinity, since its establishment, become inescapable? Why is this infinity supposed to be founded on the finite?

To make a comparison, the massive infinity of "general topology" is like the microscopic of chemistry and physics. There is an ideal, a belief, that makes the infinitesimal or the microscopic the foundational infrastructure of all things.

The infinitesimal and infinite sets of points would make an absolute foundation. This would bring logical difficulties in terms of: identity, equality, equivalence; and in terms of: inscription and differentiation; and in terms of: absolute space, relative space, ether.

For topological objects, there are also logical difficulties in terms of: presentation, object, differentiation; and in terms of: existence and coexistence. There is an analogy of Lacanian dimensions and of something calling itself "binary dimensions," which allows me to believe that Lacanian dimensions are a "beyond of the impossibility of founding."

In conclusion, it seems to me that it would be worth the effort to pay attention to the difficulties of founding things in a topology.

There is another thing that I would like to advance: it seems to me that it would be worth the effort to pay attention to drawings in topology.

Paying attention to good drawings, to bad drawings, to the absence of drawings. How is it that there are no topological drawings of the quality of Escher's drawings? Has there been a decline in drawing? Were there not more topological drawings in the 19th century than today?

What doesn't go well in drawings of mathematical topology? To be precise on this, I am going to apply myself to the following oppositions: (designating/defining), (showing/demonstrating), (possible/impossible), (complete configuration/partial configuration), (presentation/object), (example/counterexample), (object/type of object), (particular/general).

There is a declared tendency, which, in the name of an obligatory ideal, wants to do away with drawings. Drawings are suspected of distorting demonstrations. It's true. And it's true in another sense, that is to say, that demonstrations make for bad drawings and drawings without interest.

This is what goes on most of the time in geometry and in topology. Drawings are bad drawings. They are construction sites, mementos of successively introduced partial elements, indexes of used up letters, trash heaps. This is what corresponds to the fact that a demonstration is a sequence, and poses problems of existence and of construction. On the contrary, a drawing is something to be achieved, it's like a little complete theory, it's like a little complete system.

There is also the suspense of demonstrating. Once a drawing has given a complete, achieved configuration, it becomes difficult to render certain existences and certain constructions problematic. This is easier to do with a partial drawing, with an unachieved drawing. It's for this reason that lots of drawings are partial drawings. Partial drawings are usually supports of a demonstration. They are not showy and not interesting. However, in art, partial drawings, under the name "detailed studies," can be interesting.

I will mention only that there is communication between partial configuration and topological infinity.

In a demonstration, what is discussed are partial configurations and impossible configurations and especially impossible partial configurations. Drawing, on the other hand, is a showing of complete and possible configurations. Demonstration is above all the demonstration of impossibilities; showing is above all the showing of possibilities. Preoccupation with generality lends itself to showing only counterexamples. Preoccupation with generality produces especially unpleasing drawings: these are drawings that would indicate a generality of possible drawings. A drawing shows only one thing. To dwell on drawing a particular case, you have to be supported by the existence of examples.

In topology, there are examples. "General topology," on the other hand, is the land of counterexamples.

So there are bad effects of demonstrations and generalities on drawings. This is not the same thing as the difficulties proper to drawing, to presentation, to designation. Just like the difficulties linked to planar presentations of objects in three-dimensional space. Like, for example, traditionally, problems of perspective in geometry. These difficulties that I call problems of designation, or problems of presentation, are a source of failed drawings, obscure drawings. But these difficulties seem to me interesting and fertile.

A presentation of an object is much less ambitious than a general definition, than the definition of a type of object. To start with, because a presentation—or rather a designation—is particular, whereas a definition is general. Secondly, because designating a thing in three dimensions by a thing in two dimensions is less ambitious than designating and defining spatial things by language alone.

So here is the second reason that renders drawings suspect. It's that a particular designation by drawing would be too easy, and would create misunderstanding of the difficulties of general definition.

And today in mathematics, there is a foundational work that is applying itself to the potential of difficulties and problems of definition.

Difficulties and problems of designation have a very different potential, and to me this seems characteristic of topology.

Thus, I have opposed one part (definition, type of object, demonstration, generality, impossibility, counterexample, partial configuration) to another part (presentation or designation, object, showing, particularity, possibility, example, complete configuration). They're so different that one could believe that they are separated and independent; one could believe that mathematics is consecrated only to the abstraction of demonstration and definition. In part it's this that is happening today, and this makes for all the confusion. The difficulties of showing and presenting come back from time to time in demonstrations. An exact statement (*énoncé*) often has two halves, a showing half and a demonstrating half. Demonstrating impossibilities is only clear with reference to the showing of possibilities.

That is to say that demonstrating doesn't work without showing. And a definition doesn't work without designation.

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